Table 3 (*cont.*)

R (1)	N (2)	(3)	M (4)	(5)	O (6)	TI (7)	TII (8)	CI (9)	CII (10)
1920	244466	---	0	0	0	0	0	0	0
1920	244555	0	---	0	0	0	0	0	0
5760	244556	0	0	---	0	0	-144556	0	0
5760	244566	0	---	0	0	0	0	0	0
1920	244666	0	0	---	0	0	-155666	0	0
960	245555	---	0	0	0	0	0	0	0
3840	245556	---	---	---	---	144456	144456	144456	144456
5760	245566	---	0	0	0	0	0	0	0
3840	245666	---	---	---	---	145666	145666	145556	145556
960	246666	---	0	0	0	0	0	0	0
192	255555	0	---	0	0	0	0	0	0
960	255556	0	0	---	0	0	-144446	0	0
1920	255566	0	---	0	0	0	0	0	0
1920	255666	0	0	---	0	0	-144666	0	0
960	256666	0	---	0	0	0	0	0	0
192	266666	0	0	---	0	0	-166666	0	0
1	333333	---	---	---	---	---	---	111111	111111
12	333334	---	0	0	0	0	0	0	0
12	333335	0	---	0	0	0	0	0	0
12	333336	0	0	---	0	0	0	0	0
60	333344	---	---	---	---	---	---	111155	111155
120	333345	0	0	---	0	0	0	0	0
120	333346	0	---	0	0	0	0	0	0
60	333355	---	---	---	---	333344	333344	111155	111166
120	333356	---	0	0	0	0	0	0	0
60	333366	---	---	---	---	---	---	111144	111144
160	333444	---	0	0	0	0	0	0	0
480	333445	0	---	0	0	0	0	0	0
480	333446	0	0	---	0	0	---	0	0
480	333455	---	0	0	0	0	0	0	0
960	333456	---	---	---	---	---	---	111456	111456
480	333466	---	0	0	0	0	0	0	0
160	333555	0	---	0	0	0	0	0	0
480	333556	0	0	---	0	0	-333446	0	0
480	333566	0	---	0	0	0	0	0	0
160	333666	0	0	---	0	0	0	0	0
240	334444	---	---	---	---	---	---	115555	115555
960	334445	0	0	---	0	0	---	0	0
960	334446	0	---	0	0	0	0	0	0
1440	334455	---	---	---	---	---	---	115566	115566
2880	334456	---	0	0	0	0	0	0	0
1440	334466	---	---	---	---	---	---	114455	114455
960	334555	0	0	---	0	0	-334445	0	0
2880	334556	0	---	0	0	0	0	0	0
2880	334566	0	0	---	0	0	0	0	0
960	334666	0	---	0	0	0	0	0	0
240	335555	---	---	---	---	334444	334444	115555	116666
960	335556	---	0	0	0	0	0	0	0
1440	335566	---	---	---	---	334466	334466	114455	114466
960	335666	---	0	0	0	0	0	0	0
240	336666	---	---	---	---	---	---	114444	114444
192	344444	---	0	0	0	0	0	0	0
960	344445	0	---	0	0	0	0	0	0
960	344446	0	0	---	0	0	---	0	0
1920	344455	---	0	0	0	0	0	0	0
3840	344456	---	---	---	---	---	---	145556	145556
1920	344466	---	0	0	0	0	0	0	0
1920	344555	0	---	0	0	0	0	0	0
5760	344556	0	0	---	0	0	0	0	0
5760	344566	0	---	0	0	0	0	0	0
1920	344666	0	0	---	0	0	---	0	0
960	345555	---	0	0	0	0	0	0	0
3840	345556	---	---	---	---	344456	344456	145556	145666
5760	345566	---	0	0	0	0	0	0	0
3840	345666	---	---	---	---	---	---	144456	144456
960	346666	---	0	0	0	0	0	0	0
192	355555	0	---	0	0	0	0	0	0
960	355556	0	0	---	0	0	-344446	0	0
1920	355566	0	---	0	0	0	0	0	0
1920	355666	0	0	---	0	0	-344666	0	0
960	356666	0	---	0	0	0	0	0	0
192	366666	0	0	---	0	0	0	0	0
64	444444	---	---	---	---	---	---	---	---
384	444445	0	0	---	0	0	---	0	0
384	444446	0	---	0	0	0	0	0	0
960	444455	---	---	---	---	---	---	---	---
1920	444456	---	0	0	0	0	0	0	0
960	444466	---	---	---	---	---	---	444455	---
1280	444555	0	0	---	0	0	0	0	0
3840	444556	0	---	0	0	0	0	0	0
3840	444566	0	0	---	0	0	---	0	0
1280	444666	0	---	0	0	0	0	0	0
960	445555	---	---	---	---	444455	444455	444455	444466
3840	445556	---	0	0	0	0	0	0	0
5760	445566	---	---	---	---	---	---	---	---
3840	445666	---	0	0	0	0	0	0	0
960	446666	---	---	---	---	---	---	444455	---
384	455555	0	0	---	0	0	-444445	0	0
1920	455556	0	---	0	0	0	0	0	0
3840	455566	0	0	---	0	0	-444566	0	0
3840	455666	0	---	0	0	0	0	0	0
1920	456666	0	0	---	0	0	0	0	0
384	466666	0	---	0	0	0	0	0	0
64	555555	---	---	---	---	444444	444444	444444	444444
384	555556	---	0	0	0	0	0	0	0
960	555566	---	---	---	---	444466	444466	444455	444455
1280	555666	---	0	0	0	0	0	0	0
960	556666	---	---	---	---	446666	446666	444455	444466
384	566666	---	0	0	0	0	0	0	0
64	666666	---	---	---	---	---	---	444444	44444

Table 4. *The* 18 *independent fifth-order elastic constants and their equivalence for a cubic crystal*

11111 = 22222 = 33333
11112 = 11113 = 12222 = 13333 = 22223 = 23333
11122 = 11133 = 11222 = 11333 = 22233 = 22333
11123 = 12223 = 12333
11144 = 22255 = 33366
11155 = 11166 = 22244 = 22266 = 33344 = 33355
11223 = 11233 = 12233
11244 = 11344 = 12255 = 13366 = 22355 = 23366
11255 = 11366 = 12244 = 13344 = 22366 = 23355
11266 = 11355 = 12266 = 13355 = 22344 = 23344
11456 = 22456 = 33456
12344 = 12355 = 12366
12456 = 13456 = 23456
14444 = 25555 = 36666
14455 = 14466 = 24455 = 25566 = 34466 = 35566
15555 = 16666 = 24444 = 26666 = 34444 = 35555
15566 = 24466 = 34455
44456 = 45556 = 45666

Table 5. *Elastic energy* ϕ_5 *for a cubic crystal*

$C_{11111}(\eta_1^5+\eta_2^5+\eta_3^5)$
$C_{11112}[\eta_1^4(\eta_2+\eta_3)+\eta_2^4(\eta_1+\eta_3)+\eta_3^4(\eta_1+\eta_2)]$
$C_{11122}[\eta_1^3(\eta_2^2+\eta_3^2)+\eta_2^3(\eta_1^2+\eta_3^2)+\eta_3^3(\eta_1^2+\eta_2^2)]$
$C_{11123}(\eta_1^3\eta_2\eta_3+\eta_1\eta_2^3\eta_3+\eta_1\eta_2\eta_3^3)$
$C_{11144}(\eta_1^3\eta_4^3+\eta_2^3\eta_5^2+\eta_3^3\eta_6^2)$
$C_{11155}[\eta_1^3(\eta_5^2+\eta_6^2)+\eta_2^3(\eta_4^2+\eta_6^2)+\eta_3^3(\eta_4^2+\eta_5^2)]$
$C_{11223}(\eta_1^2\eta_2^2\eta_3+\eta_1^2\eta_2^3\eta_2+\eta_2^2\eta_3^2\eta_1)$
$C_{11244}[\eta_1^2\eta_4^2(\eta_2+\eta_3)+\eta_2^2\eta_5^2(\eta_1+\eta_3)+\eta_3^2\eta_6^2(\eta_1+\eta_2)]$
$C_{11255}[\eta_1^2\eta(\eta_2\eta_5^2+\eta_3\eta_6^2)+\eta_2^2(\eta_1\eta_4^2+\eta_3\eta_6^2)+\eta_3^2(\eta_1\eta_4^2+\eta_2\eta_5^2)]$
$C_{11266}[\eta_1\eta_2\eta_6^2(\eta_1+\eta_2)+\eta_1\eta_3\eta_5^2(\eta_1+\eta_3)+\eta_2\eta_3\eta_4^2(\eta_2+\eta_3)]$
$C_{11456}[\eta_4\eta_5\eta_6(\eta_1^2+\eta_2^2+\eta_3^2)]$
$C_{12344}[\eta_1\eta_2\eta_3(\eta_2^4+\eta_5^2+\eta_6^2)]$
$C_{12456}[\eta_4\eta_5\eta_6(\eta_1\eta_2+\eta_1\eta_3+\eta_2\eta_3)]$
$C_{14444}(\eta_1\eta_4^4+\eta_2\eta_5^4+\eta_3\eta_6^4)$
$C_{14455}[\eta_4^2\eta_5^2(\eta_1+\eta_2)+\eta_4^2\eta_6^2(\eta_1+\eta_3)+\eta_4^2\eta_5^2(\eta_2+\eta_3)]$
$C_{15555}[\eta_5^4(\eta_1+\eta_3)+\eta_4^4(\eta_2+\eta_3)+\eta_6^4(\eta_1+\eta_2)]$
$C_{15566}(\eta_1\eta_5^2\eta_6^2+\eta_2\eta_4^2\eta_6^2+\eta_3\eta_4^2\eta_5^2)$
$C_{44456}[\eta_4\eta_5\eta_6(\eta_4^2+\eta_5^2+\eta_6^2)]$

ing non-linear effects is appreciable, which is consistent with the point made by Chang & Barcsh (1967) that the convergence of the series expansion for the strain energy is fairly slow. The recently developed theory (Ljamov, 1972; Ljamov, Hsu & White, 1972) for the calculation of the non-linear effects in the sound velocity, can also be extended to include higher-order terms. Recent measurements in quartz (Lean & Tseng, 1970) make the inclusion of higher-order terms in calculating the amplitude of the harmonic generations pertinent.

Summary

By the use of the symmetry properties of different crystal classes, the schemes of elastic constants have been worked out to higher orders. The number of these constants agree very well with the group-theoretical predictions. These tables can provide the basis for the investigation of non-linear effects of higher orders in different solids.

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APPENDIX

Equations relating the different FOEC for trigonal and hexagonal systems

Equations (*B*1)

4444 independent
4445 = 4555 = 0
4455 = 2 . 4444
5555 = 4444

Equations (*B*2)

1444, 1445, 1455, 1555 independent
2444 = $-\frac{1}{2}$(1444 + 1455)   2445 = $-\frac{1}{2}$(3 . 1555 − 1445)
2455 = $-\frac{1}{2}$(3 . 1444 − 1455)   2555 = $-\frac{1}{2}$(1555 + 1445)
4446 = $-\frac{1}{2}$(3 . 1555 + 1445)   4456 = $\frac{1}{2}$(3 . 1444 + 1445)
4556 = $-\frac{1}{2}$(3 . 1555 + 1445)   5556 = $\frac{1}{2}$(3 . 1444 + 1445)

Equations (*B*3)

1144, 1145, 1155, 1244, 1245, 1255 independent
1446 = $\frac{1}{2}$(2 . 1145 + 3 . 1245)   1556 = $-\frac{1}{2}$(2 . 1145 + 3 . 1245)
1456 = −2 . 1144 + 2 . 1155 − 3 . 1244 + 3 . 1255
2244 = $\frac{1}{2}$(2 . 1155 − 1244 + 1255)
2255 = $\frac{1}{2}$(2 1144 + 1244 − 1255)
2446 = $\frac{1}{2}$(2 . 1145 − 1245)   2556 = $-\frac{1}{2}$(2 . 1145 − 1245)
2245 = −(1145 + 1245)   4566 = 2 . 1245
2456 = −2 . 1144 + 2 . 1155 + 1244 − 1255
4466 = $\frac{1}{2}$(2 . 1144 + 2 . 1155 + 1244 − 3 . 1255)
5566 = $\frac{1}{2}$(2 . 1144 + 2 . 1155 − 3 . 1244 + 1255)

Equations (*B*4)

1114, 1115, 1124, 1125, 1146, 1156 independent
1224 = −(2 . 1114 + 3 . 1124 − 1156)/3
1225 = −(2 . 1115 + 3 . 1125 + 1146)/3
1246 = −2(1115 + 1125)   1256 = 2(1114 + 1124)
1466 = −(1114 + 1124 − 1156)   1566 = −(1115 + 1125 + 1146)
2224 = −(1114 + 1156)/3   2225 = −(1115 − 1146)/3
2246 = −(8 . 1115 + 1146)/3   2256 = (8 . 1114 − 1156)/3
2466 = −(5 . 1114 − 1156 − 3 . 1124)/3
2566 = −(5 . 1115 + 1146 − 3 . 1125)/3
4666 = −2(1115 − 3 . 1124 − 1146)/3
5666 = 2(1114 − 3 . 1125 + 1156)/3

Equations (*B*5)

1111, 1112, 1116, 1122, 1166 independent
1126 = −1116   1226 = −1116
1222 = (8 . 1111 + 7 . 1112 − 2 . 1166)/9
1266 = 2(2 . 1111 + 1112 − 1122)
166 = −4 . 1116/3   2222 = (5 . 1111 + 1112 + 1166)/9
2226 = 1116   2266 = (16 . 1111 − 4 . 1112 − 1166)/9
2666 = −4 . 1116/3
6666 = (2 . 1111 − 5 . 1112 + 3 . 1122 + 1166)/3

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